EME 451: Introduction to CFD: Homework Set 1

Handed out 08/09/2015

Due 15/09/2015

1 Problem 1 - Consistency and Order of Accuracy(40%)

Using Taylor series analysis, show that the following numerical method

$$\frac{u_j^{n+1} - \frac{1}{2}(u_{j-1}^n + u_{j+1}^n)}{\Delta t} + \frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} = 0 \tag{1}$$

which is an approximation to the advection (or convection) equation

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0 \tag{2}$$

is inconsistent if the mesh is refined such that

$$\Delta x \to 0$$
$$\Delta t \to 0$$
$$\frac{\Delta t}{\Delta x^2} \to constant$$

What is the order of accuracy of the numerical method (both in time and space)?

2 Problem 2 - Computer Project 1: Solving a Simple ODE(60%)

We discussed in class a rather trivial ODE

$$u_t = au \tag{3}$$

In this project, you will write a code to solve this problem, making some delibrate mistakes in order to examine their effect, and in order to be convinced by the theoretical analysis.

Take $u(0) = 1$, and find the solution at $t = 2$, assuming $a = 2, a = 1/2, a = -1/2$. Use the following numerical method

$$u^{n+1} = u^n(1 + \alpha \Delta t^p) \tag{4}$$

and verify the various predictions made in the notes about the correct choice of p and α.

With the correct choices, try taking smaller and smaller Δt and see how the error behaves (start with $\Delta t = 1.0$).

Now try the following second order method. One time step consists of two stages. In the first stage, a tentative prediction is made for the solution after a half-step

$$u^* = u^n(1 + a\frac{\Delta t}{2}) \tag{5}$$

and then the full step is taken to be

$$u^{n+1} = u^n + a\Delta t u^* \tag{6}$$

By eliminating u^* and considering the the Taylor series expansion of the exact solution, verify that this method is second order accurate. This method is called the second-order Runge-Kutta method.

Compare both first and second order numerical methods with the exact solution, in terms of the amount of numerical work needed to obtain (a) 1%accuracy, (b) 0.01% accuracy.

Also, from your numerical results, verify that indeed your numerical methods are first and second order.

Extra-credit(10%): Invent or find another second order method and implement it and report how well it works.

There is no standard format for writing your report, but avoid a memory dump. Credit will not be given for a large volume of undigested results. Similar to a project in a physical laboratory a good write up will show a healthy blend of curiosity and common sense.

Feel free to use any programming language that you are most comfortable with (i.e. Matlab, C, C++, Fortran, etc). In addition, you may or may not be familiar with plotting packages (i.e. Matlab, Fieldview, Tecplot, etc) that make your presentation look professional. If you are, use them. If not, neatly hand-plotted results are acceptable.

If you are working with a partner, you need only to submit one report (Note: only for problem 2!).